Journal of Engineering Physics and Thermophysics, Vol. 66, No. 2, 1994

MATHEMATICAL MODELING OF AERODYNAMICS AND PHYSICOCHEMICAL PROCESSES IN THE FREEBOARD REGION OF A CIRCULATING FLUIDIZED BED FURNACE. 2. INTERACTION OF PARTICLES (PSEUDOTURBULENCE)

B. B. Rokhman and A. A. Shraiber

UDC 532.529:662.62:66.096.3

Formulas are obtained for determining the energy generation and dissipation rates of random (turbulent and pseudoturbulent) motion of particles due their collisions and the effect of the aerodynamic drag force. The averaged force of interparticle interaction, pseudoturbulent transfer coefficients for the "gas" of particles, and other quantities are calculated which are necessary to complete the set of equations describing the aerodynamics of the pneumatic transport zone of a circulating fluidized bed reactor [1].

In the first part of this study [1] a set of two-dimensional (axisymmetric) equations of aerodynamics for the pneumatic zone of the freeboard region of a circulating fluidized bed (CFB) reactor was obtained. The set comprises transfer equations of mass, momentum and kinetic energy of pulsation motion of gas and each fraction of char and ash particles. In the following, a method is given for calculation of some quantities that appear in these equations.

1. Averaged Force of Interparticle Collisions $\overline{C}_i$. In the calculation of $\overline{C}_i$ and in the sequel it will be assumed for simplicity that the particles are solid homogeneous spheres (with the real particle shape the problem is too complicated). Let a collision of particles i and k have taken place. After the collision the velocity of translational motion of particle i with be [2]

$$\overset{\circ}{\mathbf{V}}_i = \mathbf{V}_i + P_{ki}(\mathbf{V}_k - \mathbf{V}_i) + M_{ki}[\mathbf{e}\cdot(\mathbf{V}_k - \mathbf{V}_i]\,\mathbf{e}\,, \tag{1}$$

where $M_{ki} = Q_{ki} - P_{ki}$; $P_{ki} = 2\gamma_{ki}(1 - k_\tau)/7$; $Q_{ki} = \gamma_{ki}(1 - k_n)$; $\gamma_{ki} = m_k/(m_k + m_i)$. In Eq. (1) the particles are assumed not to rotate before the collision because, according to [1], different directions of the vectors Ω_i and Ω_k are virtually equiprobable and therefore the rotational contribution to the averaged force of the interaction is neglegible. Moreover, the pulsation components of the particle velocity before the collision will be neglected because: 1) they are much less than the averaged components [1] and 2) the vectors $\mathbf{V}'_i$ and $\mathbf{V}'_k$ can have any direction and averaging of the "pulsation" component of the effect of a single collision over all possible directions of the vectors will result in a small increment if the direction distribution of V'_p is not spherically symmetric and zero otherwise (this will be discussed later). As usual, it will be assumed that the midsection of particle i normal to the vector $(\mathbf{V}_k - \mathbf{V}_i)$ is bombarded uniformly by particles k. In order to obtain calculational formulas, the right-hand side of Eq. (1) should be averaged over all possible directions of the vector $\mathbf{e}$. The coordinates z, r, φ connected to the reactor will be replaced by the coordinates Z, R, φ (Fig. 1), where the Z axis is parallel to the vector $(\mathbf{V}_k - \mathbf{V}_i)$ lying in the plane zOr due to axial symmetry. Integration of (1) over the midsection of particle i gives

$$\langle \overset{\circ}{V}_i \rangle = \frac{1}{\pi d^2}\int_0^{2\pi} d\psi \int_0^d \overset{\circ}{\mathbf{V}}_i\, b db \left(d = OB = \frac{1}{2}(\delta_i + \delta_k)\,; \quad b = OC = d\sin\Theta \right),$$

Institute for Power Supply Problems, Kiev, Ukraine. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 66, No. 2, pp. 159-167, February, 1994. Original article submitted March 26, 1992.

and after simple calculations we arrive at

$$(\Delta \mathbf{V}_i)_z = (\Delta \mathbf{V}_i)_Z \cos\alpha\,; \quad (\Delta \mathbf{V}_i)_r = -\,(\Delta \mathbf{V}_i)_Z \sin\alpha\,;$$

$$(\Delta \mathbf{V}_i)_Z = 0.5\,(P_{ki} + Q_{ki})\,|\,\mathbf{V}_k - \mathbf{V}_i\,|\,. \tag{2}$$

Calculating the frequency of impacts according to a geometric scheme of collisions [2] and taking relations (2) into consideration, we will eventually have

$$\overline{C}_{iz} = \sum_k S_{ki}\,(\,\overline{u}_k - \overline{u}_i)\,; \quad \overline{C}_{ir} = \sum_k S_{ki}\,(\,\overline{v}_k - \overline{v}_i)\,;$$

$$S_{ki} = \frac{4.5\, m_i\, m_k \bar{\beta}_i \bar{\beta}_k\,(\delta_i + \delta_k)^2\,|\,\mathbf{V}_k - \mathbf{V}_i\,|}{\pi\, \delta_i^3\, \delta_k^3\,(m_i + m_k)} \left[\,1 - k_n + \frac{2}{7}\,(1 - k_\tau)\,\right]. \tag{3}$$

It should be noted that in [3] in a one-dimensional equation of the type of (3) the last term in square brackets was omitted, i.e., roughness of the particle surface was neglected.

2. Generation of Pseudoturbulence in a Single Impact. Let us calculate the averaged pseudoturbulent energy gained by particle i in a single collision with particle k. As usual, the pulsation components of the velocity and the rotation of the particles before the collision are neglected. It should be borne in mind that $\overline{v}_i << \overline{u}_i$. Then, only the averaged longitudinal velocities of the particles should be only considered, i.e., the vectors $\mathbf{V}_i$ and $\mathbf{V}_k$ are parallel to the z axis and the two coordinate systems in Fig. 1 ($zr\varphi$ and $ZR\varphi$) coincide.

According to (1), the r-th velocity component of particle i after the collision is equal to

$$\overset{\circ}{v}_i = M_{ki}\,[\mathbf{e}\cdot(\mathbf{V}_k - \mathbf{V}_i)\,]\,e_r\,, \tag{4}$$

and the components of the vector $\mathbf{e}$ are

$$e_z = \sin\Theta\,; \quad e_r = \cos\Theta \sin\psi\,; \quad e_\varphi = -\cos\Theta\cos\psi\,.$$

As before it will be assumed that the probability that the center of particle k will cross any element of the midsection of particle i is proportional only to the area of this element. Then, integration of the square of expression (4) over the southern hemisphere of the particle in Fig. 1 gives

$$\langle\,(\overset{\circ}{v}_i)^2\,\rangle = \frac{4}{\pi d^2}\int_0^{\pi/2} d\psi \int_0^d M_{ki}^2\,(\,\overline{u}_k - \overline{u}_i)^2\,\frac{b^2}{d^2}\left(1 - \frac{b^2}{d^2}\right)\sin^2\psi\, b\,db = \frac{1}{12}\,M_{ki}^2\,(\,\overline{u}_k - \overline{u}_i)^2\,. \tag{5}$$

It is easy to see that $<(\overset{\circ}{w}_i)^2> \;=\; <(\overset{\circ}{v}_i)^2>$ (because the problem is symmetric about the Oz axis). It should be noted that the averaged transverse velocity $<\overset{\circ}{v}_i>$ or $<\overset{\circ}{w}_i>$ is zero after the collision. Meanwhile, similarly to (2), the average longitudinal velocity of particle i is equal to

$$\langle\,\overset{\circ}{u}_i\,\rangle = \overline{u}_i + (P_{ki} + 0.5\,M_{ki})\,(\,\overline{u}_k - \overline{u}_i)\,.$$

Then the "longitudinal" component of the pseudoturbulent energy gained by particle i is

$$\langle\,(\overset{\circ}{u}_i - \langle\,\overset{\circ}{u}_i\,\rangle)^2\,\rangle = \frac{1}{12}\,M_{ki}^2\,(\,\overline{u}_k - \overline{u}_i)^2\,. \tag{6}$$

Thus, it is clear from (5) and (6) that generation of pseudoturbulent energy due to the difference of the average longitudinal velocities of the colliding particles has spherical symmetry. Because of this it is possible to suggest the hypothesis that the pseudoturbulent pulsations of the particles have spherical symmetry. Of course, for a regorous proof of the hypothesis the equation of transfer of the energy k_i of random motion of particles should be replaced

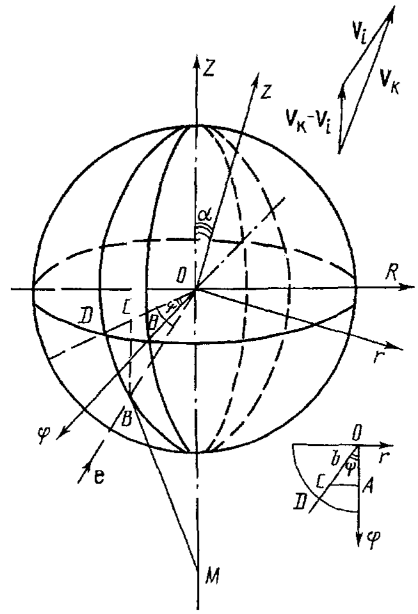

Fig. 1. Calculation of the averaged effect of a single collision.

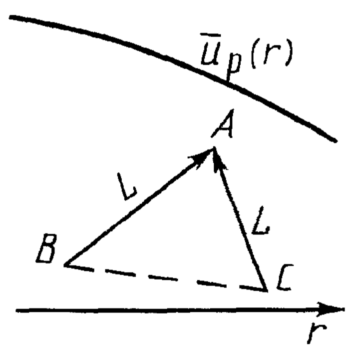

Fig. 2. Determination of the difference in averaged the longitudinal velocities of monodisperse particles.

by three equations of the components of k_i (however, this would lead to unreasonable complication of the problem, in particular, to great difficulties in calculation of the collision frequency, see below), or a precise experiment should be carried out, which is beyond the capability of present-day equipment. Therefore, we will confine ourselves to the following physical considerations: 1) if the effect of the wall is neglected, there is no reason to suggest that the dissipation of the pseudoturbulent energy will not have spherical symmetry, 2) the other terms in the equation of transfer of the energy k_i [1] should not disturb the spherical symmetry of the random motion of the particles.*

Results (5) and (6) are obtained for uniform bombardment of the midsection of particle i. In reality the cross-sectional distribution of the velocities and concentrations of the particles can be nonuniform. It is clear that this factor has no effect on $<(\overset{\circ}{w}_i)^2>$ or $<(\overset{\circ}{u}_i - <\overset{\circ}{u}_i>)^2>$. It will be assumed that within the midsection of particle i the functions $\bar{u}_k(r)$ and $\bar{\beta}_k(r)$ are linear:

$$\bar{u}_k(r) = \bar{u}_{k0} - Ar\,; \quad \bar{\beta}_k(r) = \bar{\beta}_{k0} - Br\,,$$

where the reference point for r is taken at the center of particle i. Calculations similar to those described above give

* It is most likely that turbulent pulsations of particles do not have spherical symmetry [4]. However, under the conditions in a CFB the intensity of pseudoturbulent motion is substantially higher that of turbulent motion (this will be shown later). This justifies the use of the hypothesis that about the symmetry of the entire random motion of the particles is spherical).

$$\langle \overset{\circ}{v_i} \rangle = -\frac{2d}{15\bar{\beta}_{k0}} [A\bar{\beta}_{k0} + B(\bar{u}_{k0} - \bar{u}_i)] M_{ki};$$

$$\langle (\overset{\circ}{v_i})^2 \rangle = M_{ki}^2 \left[\frac{1}{12}(\bar{u}_{k0} - \bar{u}_i)^2 + \frac{97}{7200}(Ad)^2 + \right.$$

$$\left. + \frac{97}{3600\bar{\beta}_{k0}} ABd^2 (\bar{u}_{k0} - \bar{u}_i) - \frac{4}{225\bar{\beta}_{k0}^2} (Bd(\bar{u}_{k0} - \bar{u}_i))^2 \right]. \quad (7)$$

Consequently, nonuniformity of the distributions of $\bar{u}_k$ and $\bar{\beta}_k$ leads to a directed shift of particles i along the r axis. This effect is usually small, and in the following it will be neglected; however, for a more occurate calculation this correction can be included in Eq. (5) of radial motion of the particles [1]. Estimate show that under the conditions in a CFB the terms in (7) containing A and B are 3 to 5 orders of magnitude smaller than the first term; in the following this correction will be neglected.

3. Calculation of K**. Monodisperse Material.** In accordance with the above, the following scheme of the motion and interaction of the particles will be adopted:

a) The particles participate in averaged motion with the velocities $\bar{u}_p$ and $\bar{v}_p$ and in random notion with the characteristic velocity $(\overline{V_p'^2})^{1/2} = \sqrt{2k_p}$, where $(\overline{V_p'^2})^{1/2} << \bar{u}_p$ and the distribution of the directions of the vector V_p' has spherical symmetry.

b) The main reasons for collisions are: 1) the difference of the averaged longitudinal velocities of particles that arrive at the interaction point A from some points B and C (Fig. 2); 2) The pulsations V_p'. It is assumed that $|\mathbf{BA}| = |\mathbf{CA}| = L$ and the distribution of the directions of the vectors **BA** and **CA** is equiprobable. Similarly to the molecular kinetic theory, it will be assumed that

$$L = \delta/(6\sqrt{2}\,\bar{\beta}_p). \quad (8)$$

Furthermore, it should be considered that the vector **L** connecting the points in coordinate space at which two successive impacts of a particle occurred can have a random direction. If the distribution of these directions is equiprobable (this suggestion is the most natural) the average projection of the vector **L** onto the r axis will evidently be equal to the ordinate of the center of gravity of a semisphere of radius L that "rests" on the coordinate plane $zO\varphi$ (the center of the sphere is at the coordinate origin. As is known from geometry, this ordinate is equal to $L/2$. Similarly the projection of the segment BC onto the r axis averaged over all directions of the vectors BA and CA is also equal to $L/2$. Then in formulas (5) and (6) $0.5L\partial\bar{u}_p/\partial r$ should be substituted for $\bar{u}_k - \bar{u}_i$.

The frequency N_Σ of collisions of monodisperse particles will be calculated in accordance with the scheme of the motion adopted. The difference in the averaged longitudinal velocitirs with (8) taken into account gives (see [2])

$$N_1 = \frac{1}{2\sqrt{2}} \left| \frac{\partial \bar{u}_p}{\partial r} \right|. \quad (9)$$

The frequency of impacts caused by random motion of the particles (on the analogy with the molecular kinetic theory) is equal to

$$N_2 = 12\frac{\bar{\beta}_p}{\delta}\sqrt{k_p}. \quad (10)$$

In order to find N_Σ from known N_1 and N_2, they will be summed "geometrically" with account for the fact that the difference in velocities that results in impacts N_1 is parallel to the z axis and the directions of the differences in random velocities are distributed equiprobably. For definiteness, let $N_1 > N_2$. Then N_Σ is the mean of the length of the segment BM in Fig. 1, where $OM = N_1$, $OB = N_2$, and the point B can take any position on the sphere. Obviously,

$$BM = \sqrt{N_1^2 + N_2^2 - 2N_1N_2 \sin\Theta}\,. \tag{11}$$

Integrating (11) over the sphere and considering exactly the same procedure for $N_1 < N_2$ (the point M is inside the sphere), we will have the final result

$$N_\Sigma = \begin{cases} N_1 + N_2^2/(3N_1), & N_1 > N_2; \\ N_2 + N_1^2/(3N_2), & N_1 \le N_2. \end{cases} \tag{12}$$

Thus, the rate of the pseudoturbulent energy generation is $N_\Sigma \Delta k_p$ (Δk_p is the effect of a single impact), or after substitution of (5), (6), and (8):

$$K_p^* = \frac{1}{2304}\left[\frac{\delta}{\bar\beta_p}\frac{\partial \bar u_p}{\partial r}\left(\frac{1-k_n}{2} - \frac{1-k_\tau}{7}\right)\right]^2 N_\Sigma \rho_p \bar\beta_p\,. \tag{13}$$

It is assumed in [5] that the dissipation of turbulent energy in inelastic collisions of particles is proportional to $N_\Sigma k_p(1 + k_n)^2$. Analyzing the simplest case of a central collision (the vector **e** is parallel to the difference in particle velocities), it can easily be seen that this expression includes particle elasticity incorrectly, since actually the kinetic energy loss is proportional to $f_1 = (1 - k_n^2)$. Therefore, it can be assumed to a first approximation that

$$K_p^{**} = C_1 \rho_p \bar\beta_p k_p (1 - k_n^2) N_\Sigma\,, \tag{14}$$

where C_1 is a constant to be found from comparison with experiment. However, in the general case roughness of the particle surface can also affect energy dissipation in collisions. In order to include the roughness the factor f_1 in (14) is replaced by the expression

$$f_2 = \left[1 - k_n - \frac{2}{7}(1 - k_\tau)\right]\left[1 + k_n - \frac{2}{7}(1 - k_\tau)\right] + \frac{8}{7}(1 - k_\tau)\left[1 - \frac{1}{7}(1 - k_\tau)\right]. \tag{15}$$

4. Polydisperse Material. In calculating K_i, mutual collisions of particles i and their collisions with particles of the other fractions in the flow should be considered: $K_i = K_{ii} + \sum\limits_{k \ne i} K_{ki}$. The *i-i* interaction will be described on the basis of notions developed earlier [see Eqs. (9)-(15)]. However, in calculating the effect of a single impact and the frequency N_1, L should be regarded as the free path between collisions of particles i with all the other particles (naturally, this is not the case for N_2). In a first approximation, (8) is used for calculating L but $\bar\beta_p$ and δ are replaced by the averaged equivalent particle concentration $\bar\beta_e$ and their average dimension $<\delta>$. Then $\bar\beta_p$ and δ in the square brackets in (13) are replaced by $\bar\beta_e$ and $<\delta>$ (in the other quantities the subscript i is substituted for p; moreover, relation (9) is transformed to

$$N_1 = \frac{\bar\beta_i \langle\delta\rangle}{2\sqrt{2}\,\bar\beta_e \delta_i}\left|\frac{\partial \bar u_i}{\partial r}\right| \tag{16}$$

(N_2, N_Σ, and K_{ii}^{**} are calculated from formulas similar to (10), (12), (14), (15)).

The main reason for collisions of particles from different fractions is differences in the averaged longitudinal velocities [2]. As shown by estimate, under the conditions of a CFB this factor is much more important than particle migration to the interaction point (Fig. 2). Meanwhile, the *i-k* collisions induced by random motion of particles from both fractions should be taken into consideration. With the notation

$$\bar u_{ki} = |\bar u_k - \bar u_i|\,, \quad V'_{ki} = \langle|\mathbf{V}'_k - \mathbf{V}'_i|\rangle$$

the average relative velocity V'_{ki} of particles i and k in random motion will be calculated. To do this, it will be assumed that the absolute values of random velocities of all particles i and k are $|\mathbf{V}'_i| = \sqrt{2k_i}$ and $|\mathbf{V}'_k| = \sqrt{2k_k}$

respectively, with equiprobable distribution of the directions of both vectors in space. Then, it is evident that $\mathbf{V}'_{ki}$ can be found similarly to (12):

$$V'_{ki} = \begin{cases} V'_i + V'^2_k/(3V'_i), & V'_i \geq V'_k; \\ V'_k + V'^2_i/(3V'_k), & V'_i < V'_k. \end{cases} \tag{17}$$

(If $k = i$ is assumed in (17), then $V'_{ii} = 1.33V'_i$, while the molecular kinetic theory gives $\sqrt{2}\,V'_i$ for the average relative velocity of two identical molecules. This discrepancy results naturally from the hypothesis on equality of the absolute values of velocities adopted here. However, the coefficients ($\sqrt{2}$ and 1.33) differ by just 6%, i.e., this hypothesis is quite reasonable. Now we can easily find the frequencies of the collisions

$$N_1 = E_{ki}\,\bar{u}_{ki}; \quad N_2 = E_{ki}\,V'_{ki}; \quad E_{ki} = 1.5\,(\delta_i + \delta_k)^2\,\bar{\beta}_k\,\delta_k^{-3}, \tag{18}$$

and (with (5) and (6) in view) the rate of energy generation in the k-i interaction

$$K^*_{ki} = \rho_i\,\bar{\beta}_i\,N_\Sigma\,\frac{[1 - k_n - 2\,(1 - k_\tau)/7]^2\,m_k^2}{8\,(m_i + m_k)^2}\,\bar{u}^2_{ki}. \tag{19}$$

Unlike the case of monodisperse material where determination the total energy lost by both colliding particles is sufficient for calculation of K_{p}^{**} [see (14), (15)], in the case of interaction of particles of different sizes the energy dissipation must be determined for each of the particles. To simplify the calculations it will be assumed that the vectors of particle velocities before the collision $\mathbf{V}_i$ and $\mathbf{V}_k$ are parallel to the z axis (this should not change form of the dependence of K^{**}_{ki} on k_n and k_τ). The difference in $(\mathring{V}_i)^2$ will be found for the following two cases: 1) $k_n = -1$, $k_\tau = 1$ (an absolutely elastic collision of particles with absolutely smooth surfaces, no energy loss); 2) $k_n \neq -1$, $k_\tau \neq 1$ (there is energy dissipation):

$$\Delta\,(\mathring{V}_i)^2 = (V_k - V_i)\,V_i\,\gamma_{ki}\,f_3 + (V_k - V_i)^2\,\gamma^2_{ki}\,f_4,$$

$$f_3 = 1 + k_n - \frac{2\,(1 - k_\tau)}{7}, \quad f_4 = 2 - \left[1 - k_n - \frac{2}{7}\,(1 - k_\tau)\right] \times$$

$$\times\,\frac{2}{7}\,(1 - k_\tau) - \frac{6}{49}\,(1 - k_\tau)^2 - \frac{1}{2}\left[(1 - k_n)^2 - \frac{4}{7}\,(1 - k_\tau)\,(1 - k_n)\right].$$

Assuming as before that the directional distribution of the vectors $\mathbf{V}'_i$, $\mathbf{V}'_k$ to be uniform, we have finally

$$K^{**}_{ki} = C_2\,\rho_i\,\bar{\beta}_i\,N_\Sigma\,\gamma_{ki}\,[f_3\,\sqrt{k_i\,k_{ki}} + f_4\,\gamma_{ki}\,k_{ki}], \tag{20}$$

where $\sqrt{k_{ki}}$ has the same meaning as V'_{ki} in (17), and C_2 is an empirical constant. Thus relations (16)-(20) are a solution of the present problem of polydisperse particles.

5. Additional Remarks. The equation of transfer of the energy k_i in [1] contains the term $\overline{\mathbf{F}'_i \cdot \mathbf{V}'_i}$, describing the effect of the interphase interaction force. Similarly to [4], neglecting the Safmen force here (but not in the equations of particle motion), we obtain

$$\overline{\mathbf{F}'_i \cdot \mathbf{V}'_i} = 0.75\,\xi_i\,\bar{\rho}_g\,\bar{\beta}_i\,\delta_i^{-1}\,|\bar{\mathbf{V}}_g - \bar{\mathbf{V}}_i|\,(\overline{u'_g\,u'_i} + \overline{v'_g\,v'_i} + \overline{w'_g\,w'_i} - 2\,k_i). \tag{21}$$

If pulsations of the particle velocity are expressed as $\varphi'_i = \varphi'_{it} + \varphi'_{if}$ ($\varphi = u, v, w$), it is evident that the correlations $\overline{\varphi'_g \varphi'_{ij}}$ should vanish, since turbulent pulsations of the gas velocity and pseudoturbulent pulsations of the particles are independent of each other. Consequently, the correlations in (21) include just turbulent effects and can be determined in accordance with [4].

In conclusion, it seems necessary to dwell further on three quantities appearing in the equations of [1]. In view of the dominating pseudoturbulent motion, the turbulent viscosity of the "gas" of particles is calculated by analogy with the molecular kinetic theory:

$$\nu_i = \sqrt{2k_i}\, L_i/3 . \tag{22}$$

According to [6, 7] the radial component of the Safmen force is

$$F_{Sir} = \begin{cases} 6.46\,(\mu\,\bar{\rho}_g)^{0.5}\,(\bar{u}_g - \bar{u}_i)\,(\delta_i/2)^2\,|\partial\bar{u}_g/\partial r|^{0.5}, & \mathrm{Re}_i \le 1; \quad (23)\\ 9.14\,(A_i/\mathrm{Re}_i)^{0.5}\,(\bar{u}_g - \bar{u}_i)^2\,(\delta_i/2)^2\,\bar{\rho}_g, & 1 < \mathrm{Re}_i \le 40; \quad (24)\end{cases}$$

$$A_i \equiv 0.5\,\delta_i\,|\partial\bar{u}_g/\partial r|\,(\bar{u}_g - \bar{u}_i)^{-1}$$

(if $\mathrm{Re}_i > 40$, formula (24) is used with $\mathrm{Re}_i = 40$). The rate of turbulent energy generation in tracks of rather large particles [5] is

$$\Gamma_p = 0.12 \sum_i \left\{1 - \exp\left[-(\mathrm{Re}_i/80)^2\right]\right\} \rho_i \bar{\beta}_i\,(\bar{u}_g - \bar{u}_i)^2/\tau_i . \tag{25}$$

6. Comparison with Reported Results. Expressions (13), (14), and (22) with obtained here will be compared with the reported [8-10] relations for a viscous "gas" of smooth monodisperse particles and terms in the pulsation energy transfer equation (see also [1]):

$$\mu_p = \frac{4}{5}\sqrt{\left(\frac{2}{3\pi}\right)}\,\rho_p\,\delta\,\sqrt{k_p}\,(1 - k_n)\,\bar{\beta}_p\,g;$$

$$K_p^* = \frac{4}{5}\sqrt{\left(\frac{2}{3\pi}\right)}\,\rho_p\,\delta\,\sqrt{k_p}\left(\frac{\partial\bar{u}_p}{\partial r}\right)^2 (1 - k_n)\,\bar{\beta}_p^2\,g; \tag{26}$$

$$K_p^{**} = 8\sqrt{\left(\frac{2}{3\pi}\right)}\,\frac{\rho_p\,k_p^{3/2}}{\delta}\,(1 - k_n^2)\,\bar{\beta}_p^2\,g;$$

$$\mu_p = \frac{5}{96}\sqrt{\left(\frac{2\pi}{3}\right)}\,\rho_p\,\delta\,\sqrt{k_p}; \tag{27}$$

$$\mu_p = 0.345\,\rho_p\,\delta\,\sqrt{k_p}\,\bar{\beta}_p^{2/3}\,g; \quad K_p^{**} = \frac{1}{3\sqrt{2}}\,\frac{\rho_p\,k_p^{3/2}}{\delta}\,(1 - k_n^2)\,\bar{\beta}_p\,g_1 . \tag{28}$$

Formulas (26), (27), and (28) were obtained in [8], [9] (here K_p^{**} coincides with (26)), and [10]. In (26)-(28) g is a function that depends on $\bar{\beta}_p$ and the maximum concentration β_{max} (in [9, 10] $g = [1\ (-\bar{\beta}_p/\beta_{max})^{1/3}]^{-1}$ is assumed, and the authors of [8] suggested introducing a correction factor of 0.6 for g); $g_1 = (\beta_{max}/\bar{\beta}_p)^{1/3} - 1$. Since the authors of [8-10] only consider particle collisions caused by their random motion, it is assumed in (13) and (14) that $N_\Sigma = N_2$; then (13), (14), and (22) are reduced to

$$\mu_p = \frac{1}{18}\rho_p\,\delta\,\sqrt{k_p}; \quad K_p^* = \frac{1}{768}\rho_p\,\delta\,\sqrt{k_p}\left(\frac{\partial\bar{u}_p}{\partial r}\right)^2 (1 - k_n)^2; \tag{29}$$

$$K_{\mathrm{p}}^{**} = 12\, C_1 \frac{\rho_{\mathrm{p}} k_{\mathrm{p}}^{3/2} \bar{\beta}_{\mathrm{p}}^2}{\delta} (1 - k_n^2).$$

It is clear that relations (29) predict the same dependence of the quantities considered on δ, k_{p}, ρ_{p}, and $\partial\bar{u}_{\mathrm{p}}/\partial r$ (for K_{p}^{**}, also on the reduction coefficient k_n) as (26)-(28) do. Moreover, these formulas differ among themselves in the nature of the effects of concentration and k_n [the latter refers only to (26)]. It should be noted here that the results [8-10] are obtained for rather dense systems ($\bar{\beta}_{\mathrm{p}} \gtrsim 0.1$), while the present study is concerned with gas suspension flows, where $\bar{\beta}_{\mathrm{p}} \lesssim 0.1$; this is probably the main reason for the difference in the form of the dependence of the parameters on $\bar{\beta}_{\mathrm{p}}$. If $\bar{\beta}_{\mathrm{p}} = 0.1$ and $k_n = -0.5$ are assumed, the numerical factors in formulas (26)-(29) for μ_{p} are equal to 0.073, 0.075, 0.034, and 0.056, respectively. Consequently, there is quantitative agreement (at least in the order of magnitude) of the results.

NOTATION

z, r, φ, longitudinal, radial, and transverse coordinates; **V**, velocity vector; u, v, w, its projection onto the z, r, φ, axes; **e**, unit vector directed along the line of collision to the center of particle i; m, particle mass; k_n, k_τ, reduction coefficients of the normal and tangential velocity components in collision ($k_n \leq 0$); Ω, angular velocity; C, interfraction interaction force; δ, particle size; β, true volumetric concentration; k, kinetic energy of random motion; L, mean free path of a particle between successive collisions; N, collision frequency; n, countable concentration of particles; $K = K^* - K^{**}$, K^*, K^{**}, rates of generation and dissipation of the energy of random particle motion caused by collisions; ρ, density; F, force of interphase interaction; ξ, aerodynamic drag; ν, μ, kinematic and dynamic viscosities; τ, time of dynamic relaxation; $\mathrm{Re}_i = |\mathbf{V}_{\mathrm{g}} - \mathbf{V}_i|\delta_i \nu_{\mathrm{g}}^{-1}$, Reynolds number. Subscripts: g, p, refers to gas, particles; i, k, fraction numbers; t, f, turbulent and pseudoturbulent quantities; $'$, pulsation component; $-$, $\langle\ \rangle$, averaged values; $\circ$, parameters of the particle motion after collision.

REFERENCES

1. B. B. Rokhman and A. A. Shraiber, Inzh.-Fiz. Zh., **65**, No. 5, 521-526 (1993).
2. G. L. Babukha and A. A. Shraiber, Interaction of Particles of Polydisperse Materials in Two-Phase Flows [in Russian], Kiev (1972).
3. B. B. Rokhman, V. B. Red'kin, and A. Yu. Maistrenko, in: Problems of Energy Supply [in Russian], Vol. 4, Kiev (1990), pp. 43-48.
4. A. A. Shraiber, L. B. Gavin, V. A. Naumov, and V. P. Yatsenko, Turbulent Gas Suspension Flows [in Russian], Kiev (1987).
5. L. V. Kondrat'ev, A Model and a Numerical Investigation of a Turbulent Gas Suspension Flow in a Tube , Author's Abstract of Candidate's Thesis, Leningrad (1989).
6. L. B. Gavin and A. A. Shraiber, Itogi Nauki i Tekhn., Mekh. Zhidk. Gaza, **25**, 90-182 (1991).
7. D. S. Fandy and H. A. Dwyer, J. Fluid Mech., **216**, 381-410 (1990).
8. J. Ding and D. Gidaspow, AIChE J., **35**, No. 9, 1473-1486 (1989).
9. J. L. Sinclair and R. Jackson, AIChE J., **35**, No. 4, 67-77 (1973).
10. M. A. Gol'dshtik and B. N. Kozlov, Zh. Prikl. Mekh. Tekhn. Fiz., No. 4, 67-77 (1973).